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955 L'Enfant Plaza North, S.W.
Washington, D. C. 20024

date: August 6, 1971

to: Distribution

B71 08015

from: S. N. Hou

subject: Power Spectral Density Function for Cratered and Particled Lunar Surface Elevations - Case 320

ABSTRACT

A knowledge of the roughness of the lunar terrain is essential to the design of the Lunar Rover Vehicle (LRV). The power spectral density functions (PSD's) of the terrain elevations obtained from Lunar Orbiter photographs by the United States Geological Survey (USGS) covers only a frequency range of less than 1 cycle per meter. Since the LRV design requires a PSD having a frequency coverage up to the order of 15 cycles per meter, a straight line extrapolation in logarithmic scale from the low frequency range was adopted in the present LRV design.

In this memorandum, the PSD's for lunar terrain elevations are computed, based directly on the cumulative distributions of craters and particles obtained from Apollo lunar surface photographs. Poisson processes were considered for the probability of occurrence along any linear path, it was concluded that the PSD's so derived conform reasonably well with the PSD's derived by USGS, and the high frequency portions do agree with the extrapolations used in the LRV design.

Since PSD functions represent statistical information obtained from numerous samples, other statistical conclusions can be drawn directly from the functions with no need to go through either actual or simulated samples. One example is presented in the Appendix where the expected distance for having a fixed change in elevation is calculated.

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MEMORANDUM FOR FILE

I. INTRODUCTION

Design guidelines for the Lunar Rover Vehicle (LRV) provide power spectral density functions (PSD's) to represent roughness of lunar terrain elevations. These PSD's are based on data provided by the United States Geological Survey (USGS), which only covers a frequency range of less than 1 cycle per meter (see Figure 8). However, the LRV design requires a frequency coverage of up to the order of 15 cycles per meter. Thus the technique used to develop the Design Guidelines was first to make a straight line approximation of the PSD's provided by USGS, and then extrapolate from the low frequency range to the higher frequency range.

Whether such a technique provides a reasonable PSD for the LRV design is the primary question to be addressed by this memorandum. From Reference (1), we have separate cumulative distribution curves of craters and rocks for certain specific lunar surface areas. The distribution curves are approximated by straight line logarithmic plots as shown in Figures 1 and 6, and may be formulated by Equations (II-1) and (III-1). As a typical representation of distribution curves obtained from Surveyor and Apollo observations, the distribution curves obtained from Apollo 11 photographs are chosen for numerical comparison. In the following, PSD functions will be formulated directly, and then compared to the straight line extrapolated PSD functions used in the LRV design.

II. PSD FOR LUNAR TERRAIN ELEVATIONS BY CRATER COUNT

A straight line approximation cumulative distribution of various size craters as given in Figure 1 can be expressed as

$$N = C\ S^{-m} \quad , \tag{II-1}$$

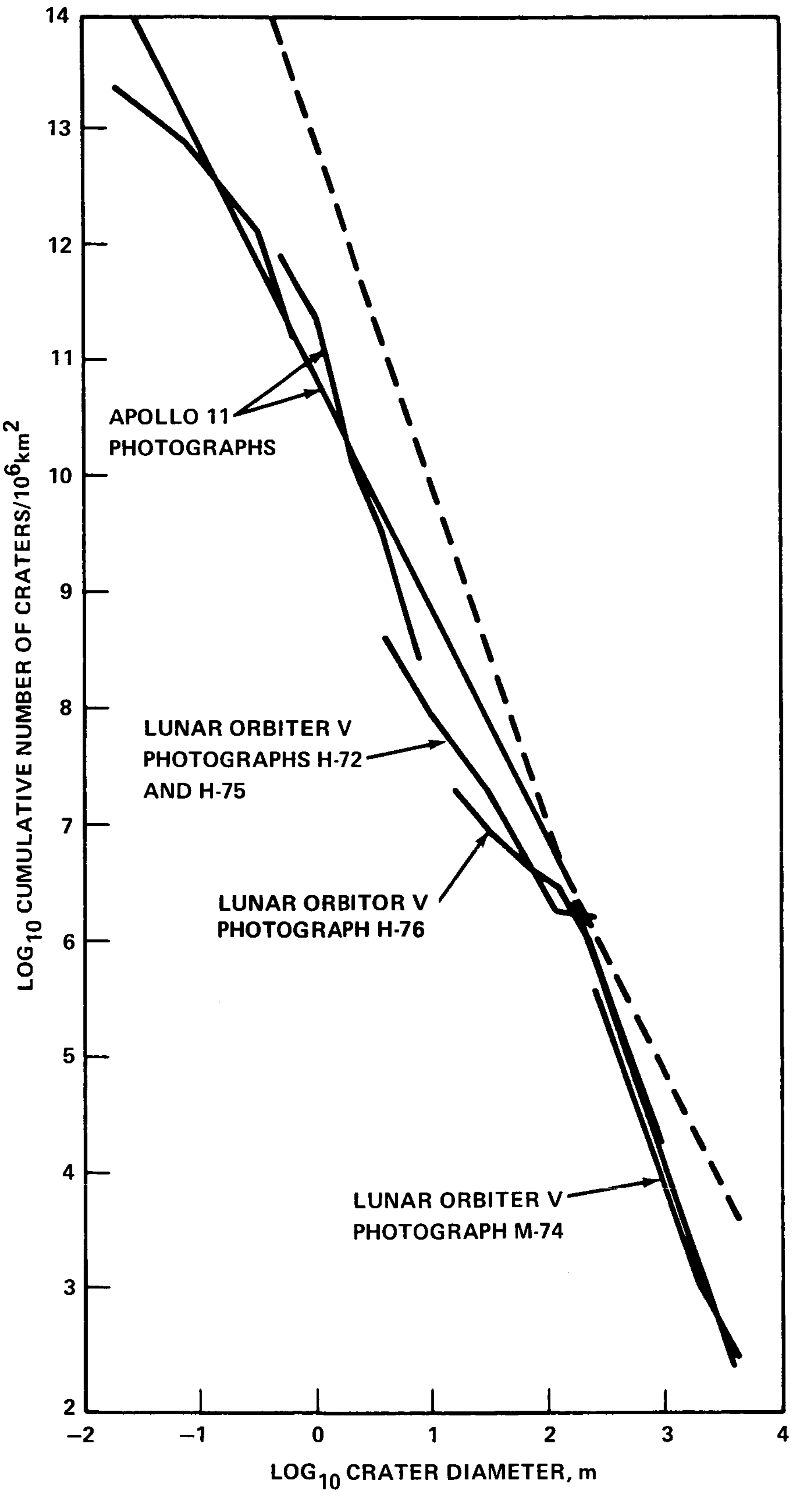

FIGURE 1 - CUMULATIVE DISTRIBUTION OF CRATERS

where C and m are constants,

N is the cumulative number of craters having size equal or greater than "S" in an area "A_C",

S is the crater diameter.

The expected number of craters, n, among size "S to S+dS" per unit area is:

$$n = \frac{1}{A_C} \left| \frac{dN}{dS} \right| dS$$

$$= \frac{Cm}{A_C} S^{-(m+1)} dS \ . \qquad \text{(II-2)}$$

In order to express the elevation effects caused by craters, a mathematical function for describing the shape of craters is needed. Physically, a lunar crater may be considered as a curved bowl encircled by an ejecta ring. By observation, the depth of a crater is roughly proportional to its diameter, and a ratio of one to six was assumed. After trying various functions, a model as shown in Figure 2 was adopted. This model has an advantage over others in that it closely approximates the shape of real craters, and can be expressed by a sinx/x function as given by Equation (II-3), where x is less than S.

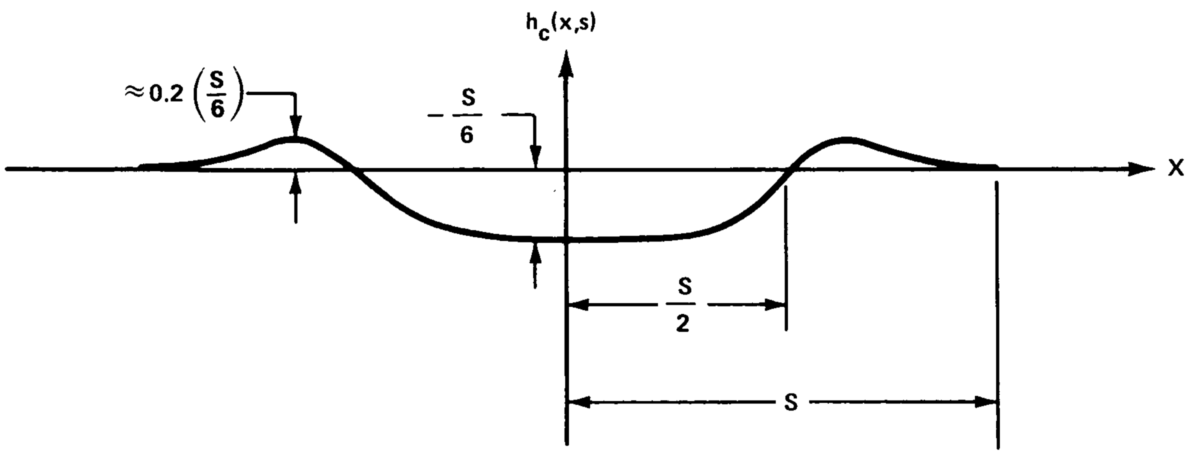

FIGURE 2 - TYPICAL CRATER PROFILE

At a given location on the lunar surface, if a crater of size "S" is to occur at a distance "x" away, (inside the shaded area of Figure 3), the effect on the surface elevation is

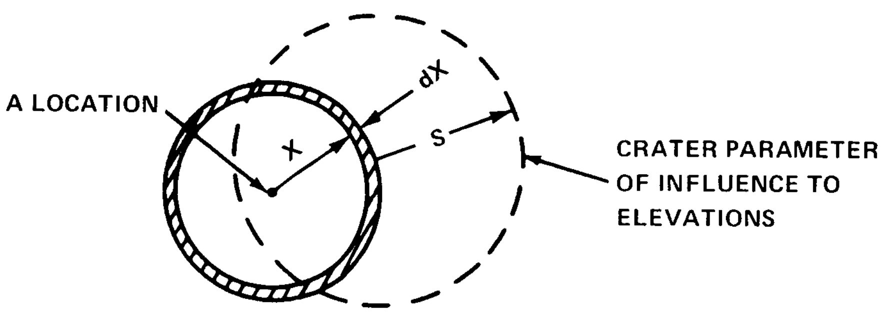

FIGURE 3- ELEVATION EFFECT OF A CRATER

$$h_C(x,S) = -\frac{S^2}{12\pi x}\,\mathrm{Sin}\,\frac{2\pi x}{S} \qquad (\text{if } 0 < x < S),$$

$$= 0 \qquad (\text{if } x > S). \tag{II-3}$$

Thus the mean square of the elevation at the given location, σ_C^2, is:

$$\sigma_C^2 = \int_{S_{min.}}^{S_{max.}} \int_0^S \left(\frac{S^2}{12\pi x}\,\mathrm{Sin}\,\frac{2\pi x}{S}\right)^2 (2\pi x dx) \left(\frac{Cm}{A_C}\,S^{-(m+1)}ds\right)$$

$$= \frac{Cm}{72\pi A}\int_{S_{min.}}^{S_{max.}} S^{3-m}\left[\int_0^S \frac{[\mathrm{Sin}\,\frac{2\pi x}{S}]^2}{x}\,dx\right] dS\ , \tag{II-4}$$

where S_{min} and S_{max} are the smallest and the largest diameter of craters under consideration. Since

$$\int_0^S \frac{[\mathrm{Sin}\,\frac{2\pi x}{S}]^2}{x}\,dx = \int_0^{2\pi} \frac{\mathrm{Sin}^2 y}{y}\,dy = 1.544\ , \qquad \text{(II-5)}$$

we have the mean square of elevation, $\sigma_C{}^2$, as follows:

$$\sigma_C{}^2 = \frac{0.02144\ Cm}{\pi A_C (4-m)}\,[S_{max}^{4-m} - S_{min}^{4-m}]\ . \qquad \text{(II-6)}$$

If we regard the statistical properties of the size distribution as given by Equation (II-1) to be true anywhere on a sizable lunar surface area, we have a stationary random process. The expected number of craters in all sizes per unit distance along a linear path is

$$n' = \left[\frac{C}{A_C}\,(S_{min}^{-m} - S_{max}^{-m})\right]^{1/2}\ . \qquad \text{(II-7)}$$

Furthermore, if we assume the probability of occurrence of craters along the path follows a Poisson process, the autocorrelation function of the lunar terrain elevation has the approximate form

$$R_C(\tau) = E[h_C(x,S),\ h_C(x+\tau,\ S)]\ ,$$

$$= \sigma_C{}^2\ e^{-n'\tau}\ , \qquad \text{(II-8)}$$

where τ is the distance between any two locations. Thus the corresponding PSD of terrain elevations, $\Phi_C(f)$, is

$$\Phi_C(f) = \int_{-\infty}^{\infty} R_C(\tau) e^{j2\pi f\tau} \, d\tau \quad ,$$

$$= \frac{\left(\frac{2\sigma^2}{n'}\right)}{1 + \left(\frac{2\pi f}{n'}\right)^2} \quad , \qquad \text{(II-9)}$$

where f is the frequency in cycles per unit distance.

In Figure 1, for the Apollo 11 case, we have

$$C = 1 \times 10^9 \quad , \quad m = 2.85 \ .$$

Thus the cumulative distribution of craters in an area $A_C = 10^{12}$ (meter)^2 is

$$N = (1 \times 10^9) \; S^{-2.85} \qquad (\text{"S" in meters}) \quad .$$

By choosing the range of crater size for consideration as

$$S_{min} = 1 \text{ meter}, \quad S_{max} = 1000 \text{ meter},$$

we have the mean square of elevation from Equation (II-6)

$$\sigma_C^{\ 2} = \frac{0.02144 \times 10^9 \times 2.85}{\pi \times 10^{12} \times (4 - 2.85)} [(1000)^{4-2.85} - (1.)^{4-2.85}]$$

$$= 4.804 \times 10^{-2} \ (\text{meter})^2$$

$$= 0.5327 \ (\text{ft})^2,$$

and the mean occurrence per unit distance by Equation (II-7)

$$n' = \left\{\frac{10^9}{10^{12}} [(1.)^{-2.85} - (1000.)^{-2.85}]\right\}^{1/2}$$

$$= 3.16 \times 10^{-2} \quad \left(\frac{\text{Craters}}{\text{Meter}}\right)$$

$$= 9.5 \times 10^{-3} \quad \left(\frac{\text{Craters}}{\text{Ft.}}\right) \quad .$$

Thus the autocorrelation function and the power spectral density function are obtained as

$$R_C(\tau) = 0.5327 \ e^{-0.0095\tau} \quad ,$$

$$\Phi_C(f) = \frac{1.122 \times 10^2}{1 + \left(\frac{f}{1.513 \times 10^{-3}}\right)^2} \quad ,$$

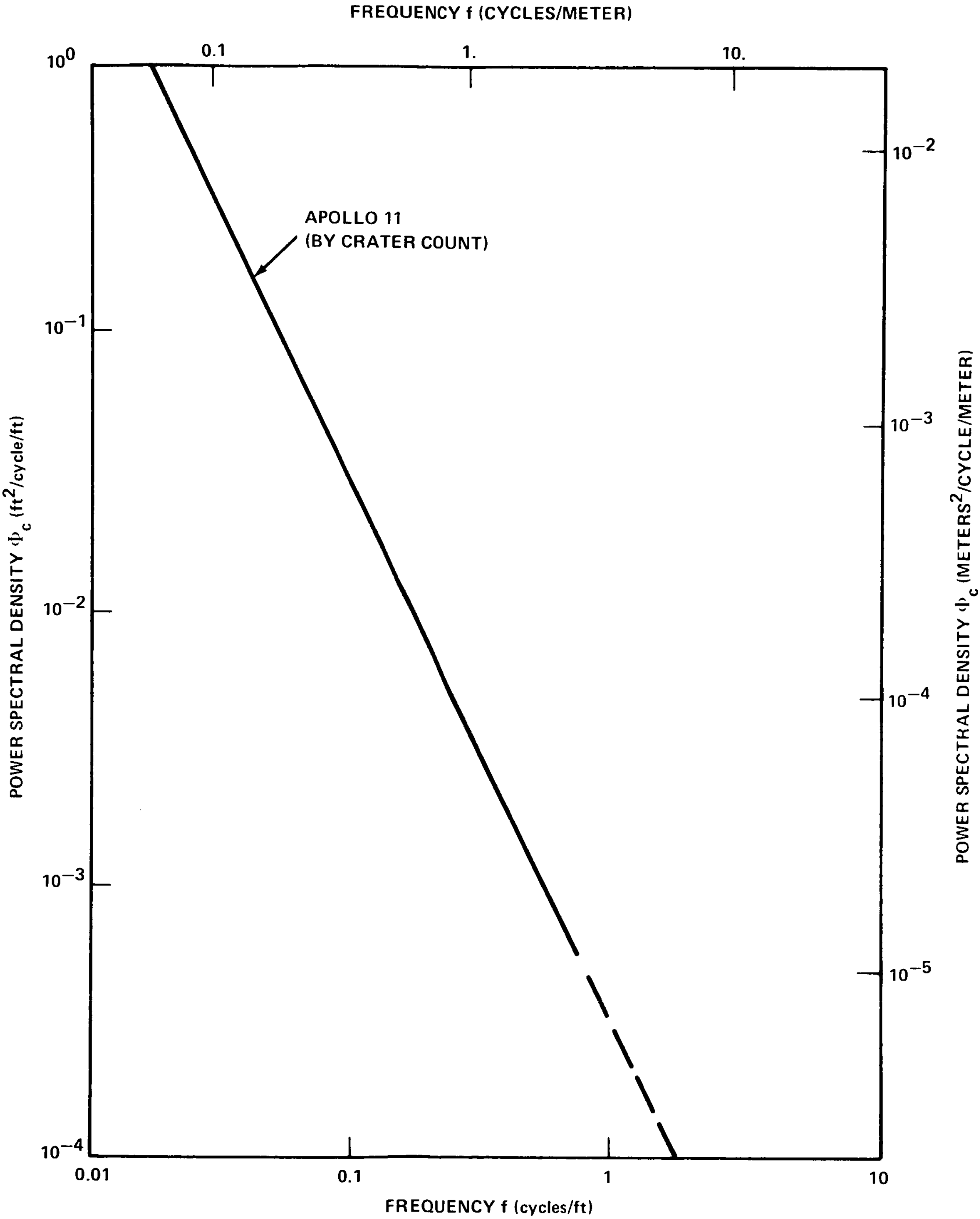

FIGURE 4 - PSD OF LUNAR TERRAIN ELEVATION BY CRATER COUNT

where τ is in units of "feet", and f is in units of "cycles per foot". The $\Phi_C(f)$ is plotted in double logrithmic scale in Figure 4; the portion drawn in solid line is the recommended range of application.

III. PSD FOR LUNAR TERRAIN ELEVATIONS BY PARTICLE COUNT

Similarly, if we are given the cumulative distribution of rocks deposited on the lunar surface, the PSD for lunar terrain elevations can be obtained by assuming that:

A. The lunar terrain elevations is a stationary random process along any linear path.

B. The rocks (particles) do not overlap one another.

C. From Reference (2), a mean "degree of burial" of 0.65 is assumed. Since there is no typical shape for rocks the model of a single rock as shown in Figure 5 is used.

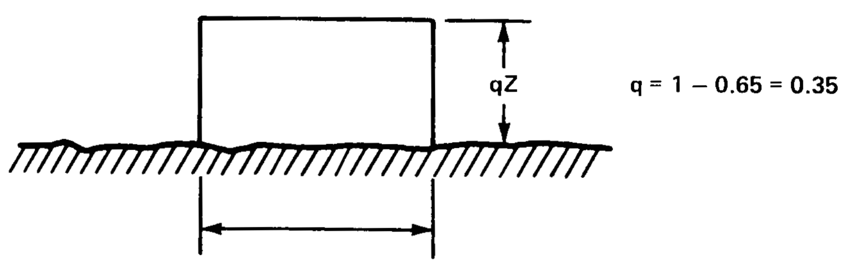

FIGURE 5 - TYPICAL ROCK MODEL

D. The probability of occurrence of rocks along a linear path follows a Poisson process.

The cumulative distribution of various size rocks as determined from Surveyor missions and Apollo 11 are given in Figure 6. These distributions can be approximated by

$$K = BZ^{-r} \quad , \qquad \text{(III-1)}$$

where "B" and "r" are constants, and "K" is the number of rocks having a size equal to or greater than "Z" in an area "A_p". Thus the expected number of rocks in size $Z \sim Z + dZ$ per unit distance along a linear path is

$$k = \left[\frac{1}{A_p} \left|\frac{dK}{dZ}\right|\right]^{1/2} dZ \quad ,$$

$$= \left(\frac{Br}{A_p}\right)^{1/2} Z^{-\frac{r+1}{2}} dZ \ . \qquad \text{(III-2)}$$

The expected number of rocks in all sizes per unit distance is

$$k' = \left[\frac{B}{A_p} \left(Z_{min}^{-r} - Z_{max}^{-r}\right)\right]^{1/2} \ . \qquad \text{(III-3)}$$

The mean square of terrain elevation is

$$\sigma_p^{\,2} = \int_{Z_{min.}}^{Z_{max.}} Z(qZ)^2 \left(\frac{Br}{A_p}\right)^{1/2} Z^{-\frac{r+1}{2}} dZ \quad ,$$

$$= \frac{2q^2}{7-r} \left(\frac{Br}{A_p}\right)^{1/2} \left[Z_{max.}^{\frac{7-r}{2}} - Z_{min.}^{\frac{7-r}{2}}\right] \ . \qquad \text{(III-4)}$$

The autocorrelation function and the PSD are obtained as before, and defined by Equations (II-8) and (II-9):

$$R_p(\tau) = \sigma_p{}^2 \, e^{-k'\tau} \tag{III-5}$$

$$\Phi_p(f) = \frac{\left(\frac{2\sigma_p{}^2}{k'}\right)}{1 + \left(\frac{2\pi f}{k'}\right)^2} \quad . \tag{III-6}$$

In Figure 6, if Apollo 11 is considered as the typical case, we have a cumulative distribution of rocks over an area $A_p = 100 \ (\text{meter})^2$ as

$$K = 0.1 \ Z^{-2.37} \quad ,$$

where Z is in meters.

By choosing the range of rock size for consideration as

$$Z_{min} = 0.02 \text{ meter,}$$

$$Z_{max} = 1. \text{ meter,}$$

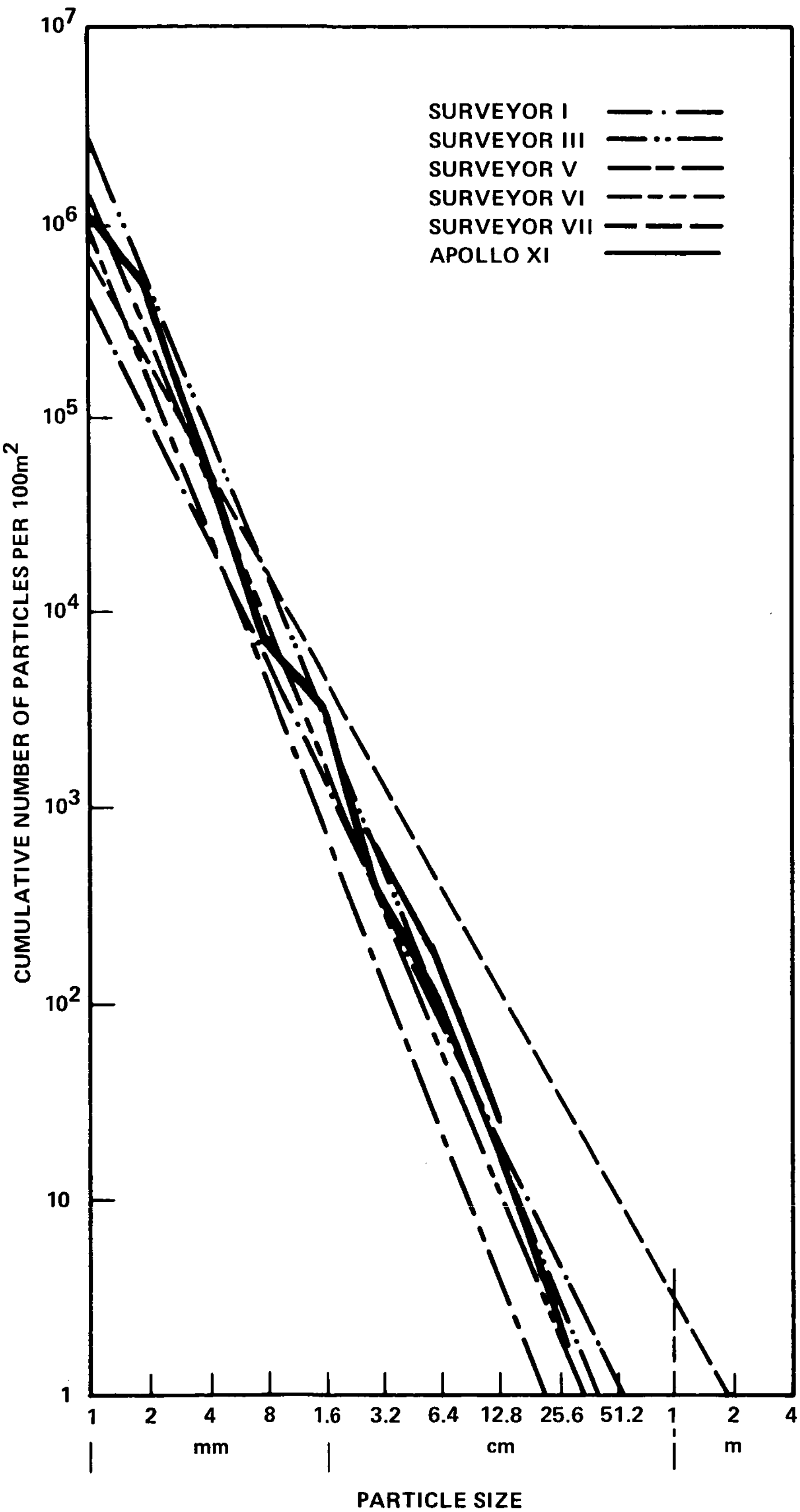

FIGURE 6 - CUMULATIVE DISTRIBUTION OF PARTICLES

we have by Equations (III-3) and (III-4),

$$k' = \left\{ \frac{0.1}{100} \left[(0.02)^{-2.37} - (1.)^{-2.37} \right] \right\}^{1/2}$$

$$= 3.26 \ \left(\frac{\text{Rocks}}{\text{meter}}\right)$$

$$= 0.978 \ \left(\frac{\text{Rocks}}{\text{Ft.}}\right) \quad ;$$

$$\sigma_p^{\ 2} = \frac{2(0.35)^2}{7-2.37} \left(\frac{0.1\text{x}2.37}{100}\right)^{1/2} \left[(1)^{\frac{7-2.37}{2}} - (0.02)^{\frac{7-2.37}{2}} \right]$$

$$= 2.57 \times 10^{-3} \ (\text{meter})^2$$

$$= 2.96 \times 10^{-2} \ (\text{ft.})^2 \ , \quad \text{and}$$

$$R_p(\tau) = (2.96 \times 10^{-2}) \ e^{-0.978\tau}$$

where τ is in unit of "feet".

Thus the PSD is computed as shown in Figure 7:

$$\Phi_p(f) = \frac{6.06 \times 10^{-2}}{1 + \left(\frac{f}{0.156}\right)^2} \quad ;$$

the portion drawn in solid line is the recommended range of application.

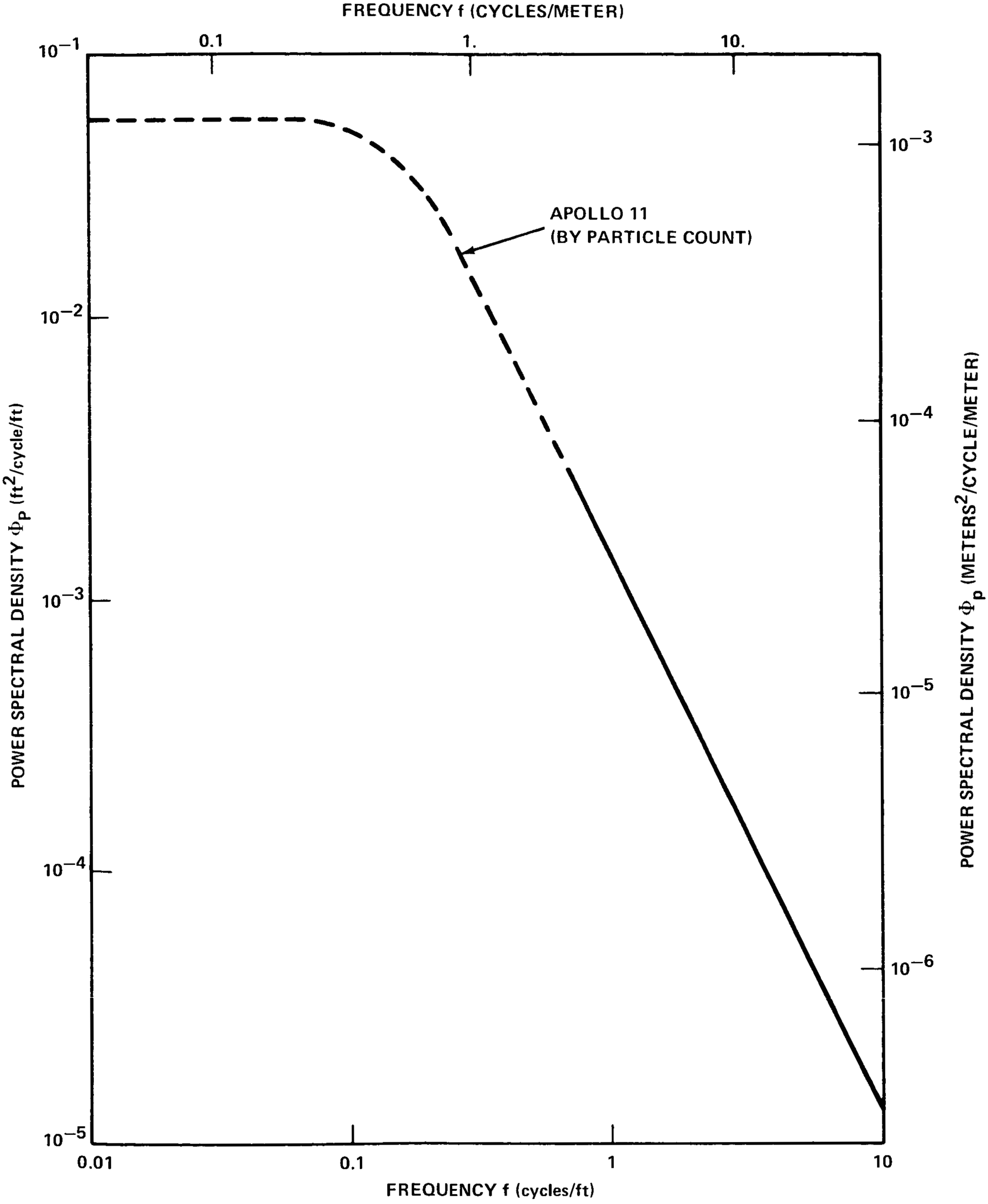

FIGURE 7 - PSD OF LUNAR TERRAIN ELEVATION BY PARTICLE COUNT

IV. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Based on separate cumulative distributions of the occurrence of craters and rocks on the lunar surface, (see Figures 1 and 6), corresponding PSD functions have been formulated. Numerical results are shown in Figures 4 and 7, both using data from Apollo 11. The results are also plotted in Figure 8 together with the smooth mare PSD's provided by USGS, to indicate the compatibility of the PSD functions.

As we know, for a large scale lunar area, craters dominate the scene. On the contrary, for a small scale lunar area, we see only rocks. Since small craters are numerous, the overlapping and scattering of ejected material will cause most of them to lose their identity and the smallest to become completely eroded. Thus the low frequency portion (i.e., large scale) of the PSD is dominated by the effects of craters, and the high frequency portion is dominated by the effects of rocks.

For applications to the LRV, the following recommendations are made:

A. Whether a lunar area is considered as large or small scale actually depends on the intended applications. Since the wheel size of the LRV is relatively small, the PSD formulated from rocks is most applicable. Note that the PSD obtained from rocks has more power than the one obtained from craters (see Figure 8) if we extend the straight line portion to cover both the low and high frequencies.

B. As shown in Figure 8, the PSD's provided by USGS do not have frequencies greater than 1 cycle per meter. A straight line extrapolation in logarithmic scale to the higher frequency range was assumed for the LRV design. In Figure 8, the formulated PSD's show a similar linear behavior, thus linear extrapolation is acceptable.

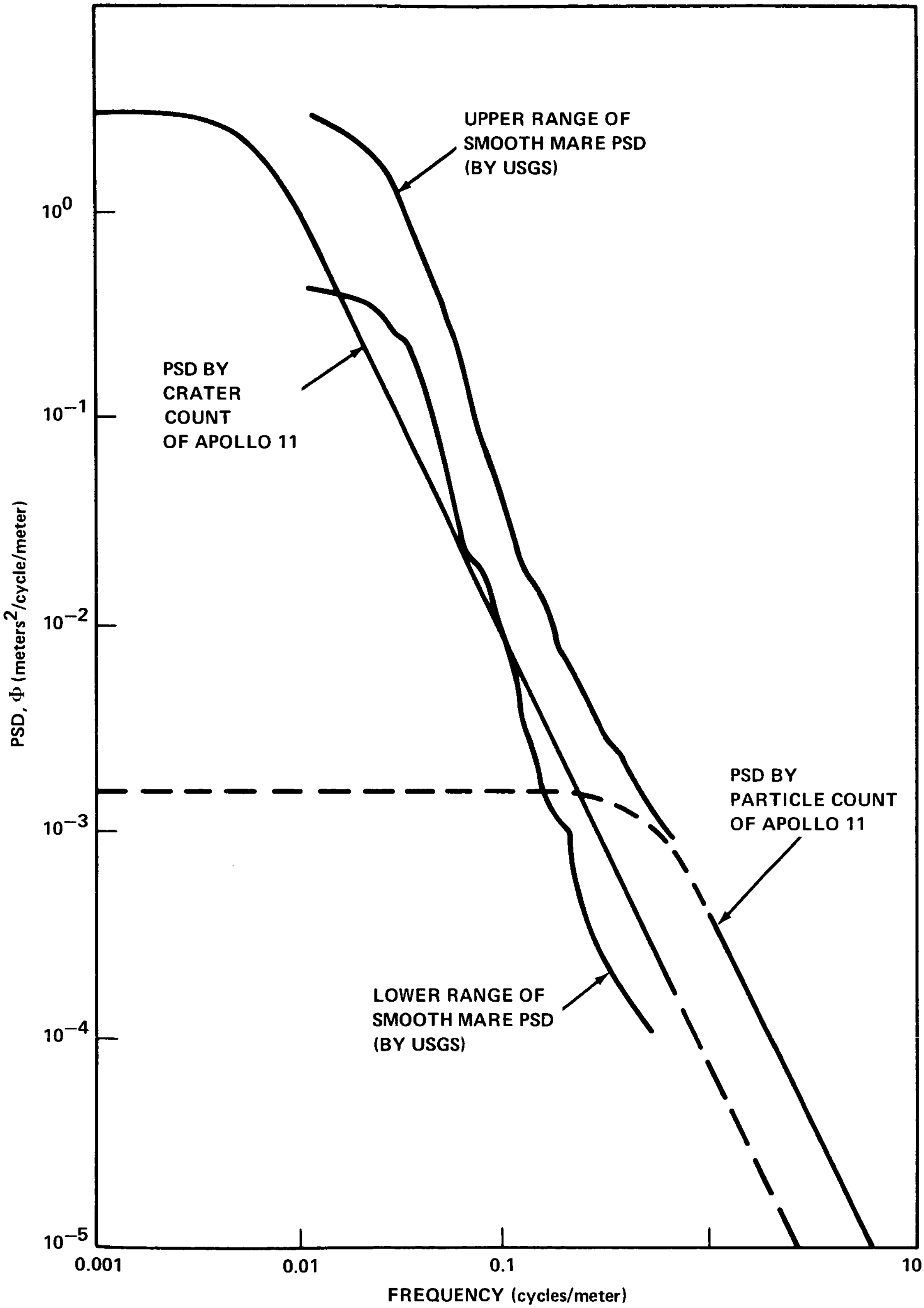

FIGURE 8 - COMPARISON OF PSD RESULTS

C. For a manned LRV, the craters and rocks greater than a certain size can be avoided. Such effects are equivalent to truncating the cumulative distribution curves of rocks and craters to a particular size, and using the truncated curves for computing PSD.

D. PSD provides useful statistical information from which other information can be obtained mathematically; the Appendix represents one example of a direct application of PSD. Using the derived PSD as shown in Figure 7, the expected distance for having a change in lunar terrain elevation, either one foot drop or one foot rise, has been computed to be 104 ft. Note that such results already represent the statistical conclusions from numerous samples, with no simulation of samples needed.

Shou-nien Hou

2031-SNH-jf S. N. Hou

Attachment
References
Appendix

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2. "Surveyor Program Results", NASA SP-184, Lunar and Planetary Programs Division, NASA, Washington, D. C., 1969.

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APPENDIX

THE MEAN DISTANCE FOR HAVING A ONE FOOT CHANGE IN ELEVATION

Suppose that the elevation h(x) of the lunar terrain along a linear path is a stationary random process having a Gaussian Distribution and zero mean, where x is the distance. This implies that the joint probability density function of h(x) and $h'(x) = \frac{dh(x)}{dx}$ take the following form[3]:

$$p(h,h') = \frac{1}{2\pi\sigma_h\sigma_{h'}} \exp\left[-\frac{1}{2}\left(\frac{h^2}{\sigma_h^2} + \frac{h^2}{\sigma_{h'}^2}\right)\right] , \tag{A-1}$$

where

$$\sigma_h^2 = \int_{-\infty}^{\infty} \Phi(f)df , \tag{A-2}$$

$$\sigma_{h'}^2 = \int_{-\infty}^{\infty} f^2\Phi(f)dw , \tag{A-3}$$

$\Phi(f)$ = PSD of the random process h(x).

This also implies that h(x) and h'(x) are uncorrelated. Thus the expected number of times that the elevation h(x) exceed a level "h(x) = a" per unit distance with positive slope is

$$N_a = \int_{0}^{\infty} h'p(a,h')dh' . \tag{A-4}$$

Substituting Equation (A-1) to (A-3) into (A-4) and denoting

$$f_0^{\,2} = \frac{\sigma_h^{\,2}}{\sigma_{h'}^{\,2}} \quad , \tag{A-5}$$

$$T_0 = \frac{1}{f_0} \quad , \tag{A-6}$$

we have

$$N_a = \frac{1}{T_0} \exp\left(- \frac{a^2}{2\sigma_h^{\,2}}\right) \quad . \tag{A-7}$$

For a distance x, the total number of times that h(x) will exceed the level "a" is

$$N_a x = \frac{x}{T_0} \exp\left(- \frac{a^2}{2\sigma_h^{\,2}}\right) \quad . \tag{A-8}$$

Thus the expected distance that h(x) exceeds the level "a" only one time is

$$x_a = T_o \exp\left(\frac{a^2}{2\sigma_h^{\,2}}\right) \quad . \tag{A-9}$$

This also implies that the probability that h(x) will exceed the level "a" in a unit distance is

$$g(a) = \frac{1}{x_a} = \frac{1}{T_o} \exp\left(- \frac{a^2}{2\sigma_h^2}\right) . \qquad \text{(A-10)}$$

Note that Equations (A-7) and (A-10) are similar, we can only apply Equation (A-10) when N_a is less than unity. Since h(x) is a Gaussian process, the occurrence of h(x) that will exceed a level $h(x) = a_1$ and the occurrence of h(x) that will exceed another level $h(x) = a_2$ are independent events, the probability for successive occurrences of these two events is

$$g(a_1, a_2) = g(a_1)g(a_2)$$

$$= \frac{1}{T_o^2} e^{-\frac{1}{2\sigma_h^2}[a_1^2+a_2^2]} . \qquad \text{(A-11)}$$

Now, using units of feet and setting

$$a_1 = a \quad , \quad a_2 = a - 1 ,$$

the frequency of occurrence for either a one-foot-drop or a one-foot-rise is

$$G = \frac{2}{T_o^{\,2}} \int_0^\infty e^{-\frac{1}{2\sigma_h^{\,2}}\,[a^2 + (a-1)^2]}\, da \quad ,$$

$$= \frac{2}{T_o^{\,2}} \int_0^\infty e^{-\frac{1}{2\sigma_h^{\,2}}\,[2(a^2 - a + \frac{1}{4}) + \frac{1}{2}]}\, da \quad ,$$

$$= \frac{2}{T_o^{\,2}}\, e^{-\frac{1}{4\sigma_h^{\,2}}} \int_0^\infty e^{-\frac{(a - \frac{1}{2})^2}{\sigma_h^{\,2}}}\, da \quad ,$$

$$= \frac{2}{T_o^{\,2}}\, e^{-\frac{1}{4\ h^2}} \int_{-\frac{1}{2}}^\infty e^{-(\frac{x}{\sigma_h})^2}\, dx \quad . \qquad \text{(A-12)}$$

Since the error function is defined as

$$\mathrm{Erf}(z) = \frac{1}{\sqrt{\pi}} \int_{-z}^{z} e^{-t^2}\, dt \quad , \qquad \text{(A-13)}$$

by changing the integration variable "x" in Equation (A-12) to "t" and letting

$$t = \frac{x}{\sigma_h} \quad , \qquad \text{(A-14)}$$

we obtain

$$G = \left(\frac{2}{T_o^{\,2}} \, e^{-\frac{1}{4\sigma_h^{\,2}}} \right) \sigma \int_{-\frac{1}{2\sigma}}^{\infty} e^{-t^2} \, dt \quad ,$$

$$= \left(\frac{2}{T_o^{\,2}} \, e^{-\frac{1}{4\sigma_h^{\,2}}} \right) \left(\frac{\sigma\sqrt{\pi}}{2} \, [1 + \mathrm{Erf}(\tfrac{1}{2\sigma})] \right) \quad ,$$

$$= \frac{\sigma\sqrt{\pi}}{T_o^{\,2}} \, e^{-\frac{1}{4\,h^2}} \, [1 + \mathrm{Erf}(\tfrac{1}{2\sigma})] \quad . \tag{A-15}$$

Thus the expected distance before having a plus or minus one foot change in elevation is,

$$X = \frac{1}{G} \quad , \tag{A-16}$$

$$X = \frac{T_o^{\,2} e^{\frac{1}{4\sigma_h^{\,2}}}}{\sigma\sqrt{\pi}[1 + \mathrm{Erf}(\frac{1}{2\sigma_h})]} \quad . \tag{A-17}$$

Now, using the PSD in Figure 7, which was obtained from Apollo 11 data, if the frequency range we are interested in is $0.01 \sim 4.0$ $(\frac{\text{cycles}}{\text{ft.}})$ and using only the straight line portion of $\Phi(f)$, which is

$$\Phi(f) = 1.475 \times 10^{-3} f^{-2} \quad ,$$

we have

$$\sigma_h^{\ 2} = \int_{0.01}^{4.0} 1.475 \times 10^{-3} f^{-2}\, df = 1.471 \times 10^{-1} \ (\text{ft}^2) \quad ,$$

$$\sigma_h = 0.384 \ (\text{ft}) = 4.61 \ (\text{in}) \quad ,$$

$$\sigma_{h'}^{\ 2} = \int_{0.01}^{4.0} f^2 \ \Phi(f) df = 5.88 \times 10^{-3} \ (\text{cycles})^2 \quad ,$$

$$f_o^{\ 2} = \frac{\sigma_{h'}^{\ 2}}{\sigma_h^{\ 2}} = 0.04 \ \left(\frac{\text{cycle}}{\text{ft}}\right)^2 \quad ,$$

$$f_o = 0.2 \ \left(\frac{\text{cycle}}{\text{ft}}\right) \quad ,$$

$$T_o = \frac{1}{f_o} = 5.0 \ \left(\frac{\text{ft}}{\text{cycle}}\right) \quad .$$

Thus the expected distance for having a one foot change in elevation is

$$x = \frac{(5)^2 \; e^{\frac{1}{4 \times 0.1471}}}{0.384 \times \sqrt{\pi} \times [1 + \mathrm{Erf}(\frac{1}{2 \times 0.384})]} = 104 \text{ ft.}$$

Subject: Power Spectral Density Function for Cratered and Particled Lunar Surface Elevations - Case 320

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